Imperfections Influence Modeling upon Eigenvibrations in a Convex Piezoelectric Plate of AT-cut

Oleksandr Yu. Shmaliy* and Yuriy S. Shmaliy⋆
*Mathematical Dept., Ukrainian Engineering Pedagogical Academy (UEPA) Kharkiv, Ukraine
Email: oleksandr@shmaliy.com
⋆FIMEE, Guanajuato University, Salamanca, Gto, 36730, Mexico

***Abstract*— A discrepancy is analyzed between the measurements and predictions of the amplitude distributions of eigenvibrations in a trapped-energy convex piezoelectric plate of thickness-shear vibrations that is in the rotation of a measured picture of the amplitude distribution regarding the plate axes, unlike the strongly symmetric case associated with a spherical surface. Such a rotation is neatly seen in the measurements obtained by a highly sensitive laser speckle method. We show that a possible source of the asymmetry and displacements is in the imperfections of a convex surface of a manufactured plate.**

I. Introduction

It is well known that measurements of the amplitude distribution over a vibrating piezoelectric plate of thickness-shear vibrations typically demonstrate a picture that is rotated regarding the plate coordinates. An example can be found in measurements provided by the laser speckle method described in [1]. Such a rotation cannot be explained by the traditional models implying symmetry of a plate. In this paper, we show that the measured asymmetric picture can be explained by the model if we suppose that the convex surface has small imperfections. We model these imperfections with an elliptic convexity arbitrary oriented for the resonator axes.

II. Object of Investigations

Let us examine a piezoelectric plate with a one-sided elliptical convexity. For better understanding, we suppose that the plate is manufactured from a big crystalline ellipsoid as shown on Fig. 1. The main parameters of the ellipsoid are the radii R_1, R_2, and R_3 and an angle $alpha$ between the main axes of an ellipsoidal surface and the coordinate system axes (Fig. 1).

This model differs from those already investigated in [2], [3] by the ellipsoidal geometry and arbitrary oriented ellipsoidal axes. In the classical spherical case, there is no idea about the axes orientation because the radii are equal in all directions. In the ellipsoidal case, one cannot drop this principle factor. Otherwise, the model lacks information about the direction of deformation that makes it meaningless for our purposes.

For the ellipsoidal case, the frequency spectrum, eigenvibrations, and distributions of the amplitudes were obtained in [4]. Following [3], the vibration frequency spectrum of a resonator

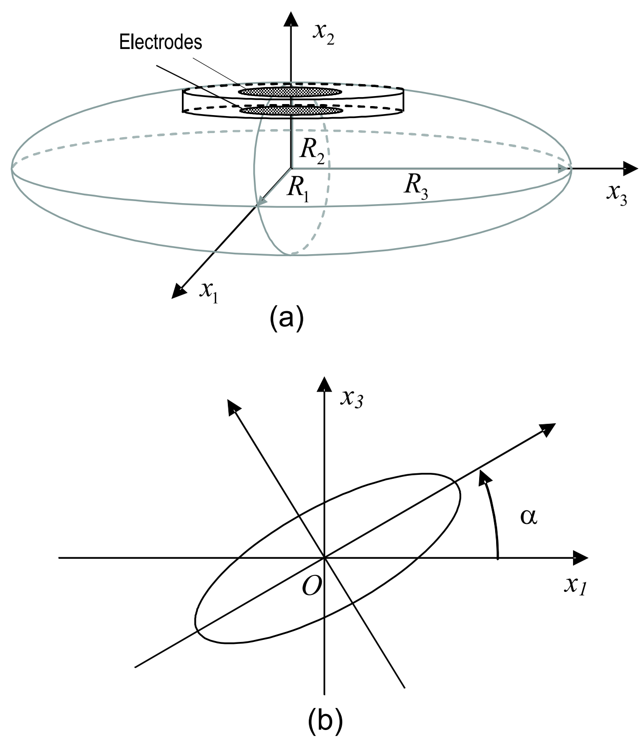

Fig. 1. The model of a crystalline piezoelectric plate of an ellipsoidal shape: (a) a main view and (b) a plane section, where α is an angle between the main axis of an ellipsoidal surface and the coordinate system axis.

is found by a solution of the effective differential equation in the self axes x and y of the tensor $d_{\alpha\beta}$, that is

$$\begin{gathered} -d_1\frac{\partial^2\Psi}{\partial x_1^2} - d_2\frac{\partial^2\Psi}{\partial x_3^2} \\ +\omega_{nF}^2\left[1+\frac{R_3}{LR_1^2R_2^2}\left(a_{11}x_1^2+a_{12}x_1x_3+a_{13}x_3^2\right)\right] \\ \times\Psi(x_1,x_3)=\omega^2\Psi(x_1,x_3)\,. \end{gathered} \tag{1}$$

where d_1 and d_2 are eigenvalues of a tensor $d_{\alpha\beta}$ coupled with the piezoelectric constants [2], ω_{nF} is a fundamental frequency of n-overtone, and

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$$a_{11} = R_3^2 \cos^2\alpha + R_1^2 \sin^2\alpha\,,$$
$$a_{12} = (R_3^2 - R_1^2)\sin 2\alpha\,,$$
$$a_{13} = R_3^2 \sin^2\alpha + R_1^2 \cos^2\alpha \qquad (2)$$

We now substitute $x_1 = \sqrt{d_1}\tilde{\xi}$ and $x_3 = \sqrt{d_2}\tilde{\eta}$ and arrive at the equation

$$\frac{\partial^2\Psi^{(0)}}{\partial\tilde{\xi}^2} + \frac{\partial^2\Psi^{(0)}}{\partial\tilde{\eta}^2} + \left[\omega^2 - \omega_{nF}^2 - \frac{\omega_{nF}^2 R_3}{LR_1^2R_2^2} \times \left(a_{11}d_1\tilde{\xi}^2 + a_{12}\sqrt{d_1d_2}\tilde{\xi}\tilde{\eta} + a_{13}d_2\tilde{\eta}^2\right)\right] \times\Psi^{(0)}(\tilde{\xi},\tilde{\eta}) = 0\,. \qquad (3)$$

To exclude the product of $\tilde{\xi}\tilde{\eta}$ in (3), we rotate the model on an angle β introducing

$$\begin{cases} \tilde{\xi} = \xi\cos\beta + \eta\sin\beta \\ \tilde{\eta} = -\xi\sin\beta + \eta\cos\beta \end{cases}, \qquad (4)$$

where

$$(a_{11}d_1 - a_{13}d_2)\sin 2\beta + a_{12}\sqrt{d_1d_2}\cos 2\beta = 0\,,$$
$$\beta = \frac{1}{2}\arctan\frac{a_{12}\sqrt{d_1d_2}}{a_{13}d_2 - a_{11}d_1}\,. \qquad (5)$$

After the manipulations, we obtain

$$\frac{\partial^2\Psi^{(0)}}{\partial\xi^2} + \frac{\partial^2\Psi^{(0)}}{\partial\eta^2} + \left[\omega^2 - \omega_{nF}^2 - \frac{\omega_{nF}^2R_3}{LR_1^2R_2^2}\left(c_{11}\xi^2 + c_{12}\eta^2\right)\right] \times\Psi^{(0)}(\xi,\eta) = 0\,, \qquad (6)$$

where

$$c_{11} = a_{11}d_1\cos^2\beta - \frac{1}{2}a_{12}\sqrt{d_1d_2}\sin 2\beta + a_{13}d_2\sin^2\beta\,. \qquad (7)$$

$$c_{12} = a_{11}d_1\sin^2\beta + \frac{1}{2}a_{12}\sqrt{d_1d_2}\sin 2\beta + a_{13}d_2\cos^2\beta\,. \qquad (8)$$

A solution of (6) can now be found in the form of $\Psi^{(0)}(\xi,\eta) = f(\xi)g(\eta)$ via

$$f''(\xi)g(\eta) + f(\xi)g''(\eta) + \left[\omega^2 - \omega_{nF}^2 - \frac{\omega_{nF}^2R_3}{LR_1^2R_2^2}\left(c_{11}\xi^2 + c_{12}\eta^2\right)\right] \times f(\xi)g(\eta) = 0 \qquad (9)$$

and we arrive at

$$\Psi^{(0)}(\xi,\eta) = \left(\frac{\sqrt{b_1}}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^m m!}}\left(\frac{\sqrt{b_2}}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^p p!}} \times e^{-\frac{\sqrt{b_1}}{2}\xi^2}H_m(\xi b_1^{1/4})e^{-\frac{\sqrt{b_2}}{2}\eta^2}H_p(\eta b_2^{1/4})\,, \qquad (10)$$

$$\omega_{nmp}^2 = \omega_{nF}^2 + 2\sqrt{b_1}\left(m + \frac{1}{2}\right) + 2\sqrt{b_2}\left(p + \frac{1}{2}\right)\,, \qquad (11)$$

where $m, p = 1, 2, 3, \ldots$ and

$$b_1 = \frac{\omega_{nF}^2R_3}{LR_1^2R_2^2}c_{11}, \qquad (12)$$

$$b_2 = \frac{\omega_{nF}^2R_3}{LR_1^2R_2^2}c_{12}\,. \qquad (13)$$

If to introduce now two dimensionless variables $\varepsilon = d_2/d_1$ and $\delta = R_2/R_1$ dependent on the anisotropy and properties of piezoelectric material and geometry of the boundary surface, respectively, then the frequency spectrum of a piezoelectric plate can be found to posses a traditional form of

$$\omega_{nmp}^2 = \omega_{nF}^2 + 2\omega_{nF}\sqrt{\frac{R_3d_1}{LR_2^2}} \times\left[\sqrt{c(\beta)}\left(m + \frac{1}{2}\right) + \sqrt{c\left(\beta + \frac{\pi}{2}\right)}\left(p + \frac{1}{2}\right)\right] \qquad (14)$$

where $n = 1, 2, 3, \ldots$, the auxiliary functions are

$$c(\beta) = \gamma(\alpha)\cos^2\beta + \frac{\sqrt{\varepsilon}}{2}\left(1 - \delta^2\right)\sin 2\alpha\sin 2\beta + \varepsilon\gamma\left(\alpha + \frac{\pi}{2}\right)\sin^2\beta\,, \qquad (15)$$

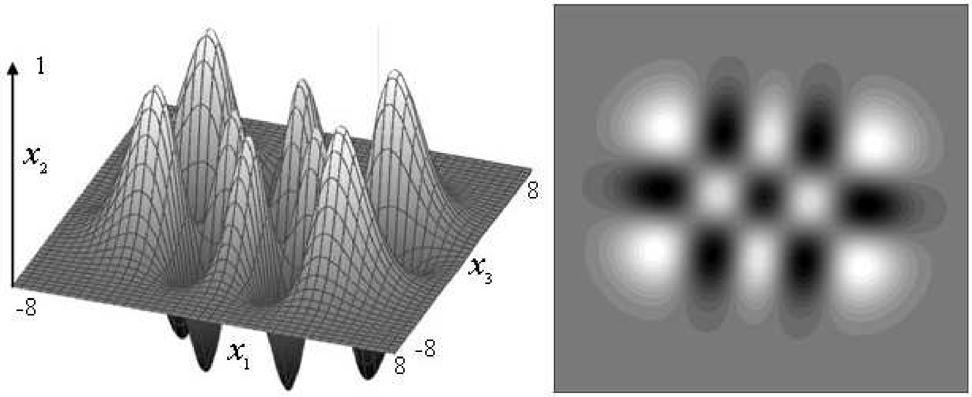
$$\gamma(\alpha) = \sin^2\alpha + \delta^2\cos^2\alpha \qquad (16)$$

and α is an angle between the axes of the tensor $d_{\alpha\beta}$ and the ellipsoidal boundary surface.

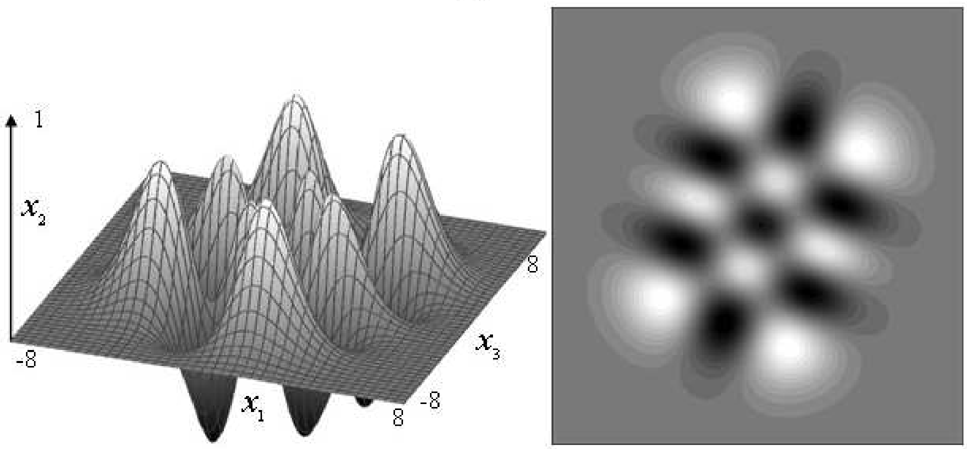
Transforming (5), by (2) and (14), finally specifies an unknown coefficient

$$\beta = \frac{1}{2}\arctan\frac{\sqrt{\varepsilon}\left(1 - \delta^2\right)\sin 2\alpha}{\gamma(\alpha) - \varepsilon\gamma\left(\alpha + \frac{\pi}{2}\right)}\,. \qquad (17)$$

We notice that (13)–(15) describe the dependence of the self-frequencies ω_{nmp} on the anisotropy parameters ε and δ as well as on the inter orientation of the self-axes of the piezoelectric plate and its boundary surface.



(a)



(b)

Fig. 2. Distributions of the normalized amplitudes for $nmp = 342$, C-mode: (a) $R_1 = 249,6$ mm, $R_2 = R_3 = 260$ mm, and (b) $R_1 = 247$ mm, $R_2 = R_3 = 260$ mm, $\alpha = 89°$.

III. Numerical Results

The distributions u_{nmp} of the normalized amplitude of AT-cut resonator are shown on the Fig. 2 for the spherical case and rotated ellipsoid regarding the vibrations of C-mode with the index $nmp = 342$. Special attention should be focused on the influence of radii deviation to the ellipsoidal case on eigenvibrations. If the main axes direction of an ellipsoidal surface is not coincided with the coordinate system axes, one must introduce an angle α . This angle affects strongly the distribution of the amplitudes by causing the rotation of the distribution area on say angle β.

Let us examine Fig. 2 in detail. The picture demonstrates distributions of the amplitude for the ellipsoidal case with a radius $R_1 = 249.6$ mm that is 4% lesser then R_2 and R_3. As can be seen, the rotation in Fig. 2a is insignificant, albeit not negligible, that corresponds to $\alpha = 89°$ close to normal direction. The strong rotation case is shown in Fig. 2b. What makes the rotation shown Fig. 2b to be so large? Simulation shows that the cause is only in 1% change in a radius R_1, if R_2, R_3, and α are constant.

The more detail explanation of how much the picture may be rotated for a particular angle gives a plot shown in Fig. 3. Here the dependence of an angle β on an angle α is shown for various ratios of R_1/R_3. It is seen that β depends both on the plate-cut orientation and plate geometry. Two bold lines (dashed and solid) on Fig. 3 correspond to the vibrations on Fig. 2a and Fig. 2b, respectively. We notice that an angle $\alpha = 89°$ was chosen intentionally to demonstrate the biggest rotation effect, by varying R_1. Another values of α do not affect the distribution as dramatically as $\alpha = 89°$. It does not mean, however, that all of the possible cases are covered by Fig. 3. For the allowed angular range of the AT-cut, one can find similar pictures exhibiting rotations of the amplitude distributions meaning that the effect is fundamental.

Let us recall that the rotation angle β depends on the vibration mode (A, B or C), main harmonic mode ($n = 1$, $n = 3$, $\dots$) and does not depend on the anharmonic modes represented by the coefficients m and p. By virtue of that and

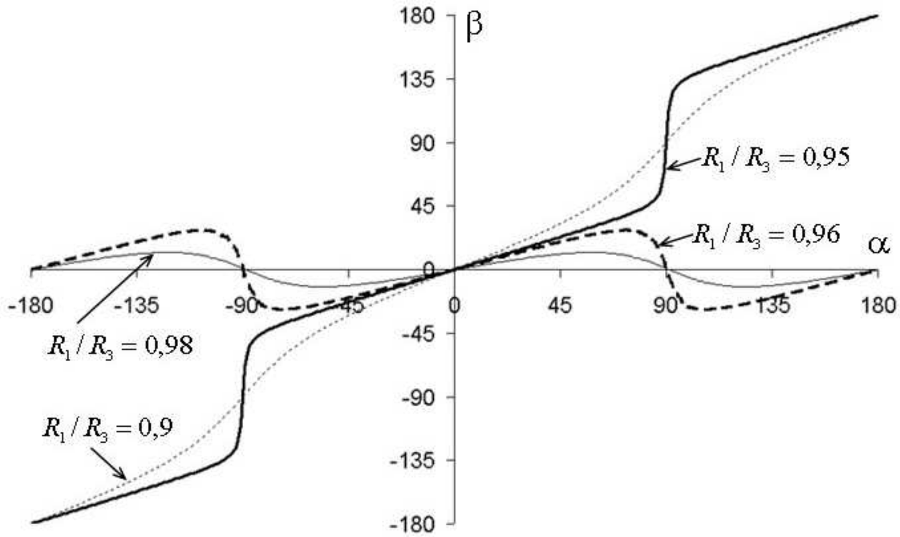


Fig. 3. Dependence of an angle β on an angle α for different values of the ratio R_1/R_3.

the above analysis, Fig. 2a reminiscent of Fig. 23 in [1] and Fig. 2b looks very closely to Fig. 24 in [1]. Note that both these pictures were measured experimentally for AT-cut. We thus infer that the model discussed reveals a capability of explaining subtle effects in measurements of the amplitude distributions in piezoelectric resonators of thickness-shear vibrations. It thus can be used to solve an inverse problem, namely to predict possible imperfections in the plate surface via measurements of rotations in the amplitude distributions.

IV. Conclusions

In this paper, an analysis is given of eigenvibrations of a piezoelectric resonator with one-sided elliptical convexity. The quartz plate of AT-cut was considered. We discussed variations in the amplitude distributions caused by an elliptical shape specified by the radii ratio R_1/R_3 and angle α between the main axis of an ellipsoidal surface and the coordinate system axis. In the considered case, the amplitude distribution rotates dramatically when $R_1 = 249,6$ mm, $R_1 = 247$ mm, $R_2 = R_3 = 260$ mm, and $\alpha = 89°$. The latter means that the AT-cut plate is highly sensitive to even insignificant imperfections in the surface. We notice that the other cuts of piezoelectric resonators can reveal even higher sensitivity and, as a result, much stronger effect in the area of deformation [5], [6]. These cuts are currently under investigations aimed at finding analytical relations between the imperfections in the surface and distributions of the amplitudes.

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